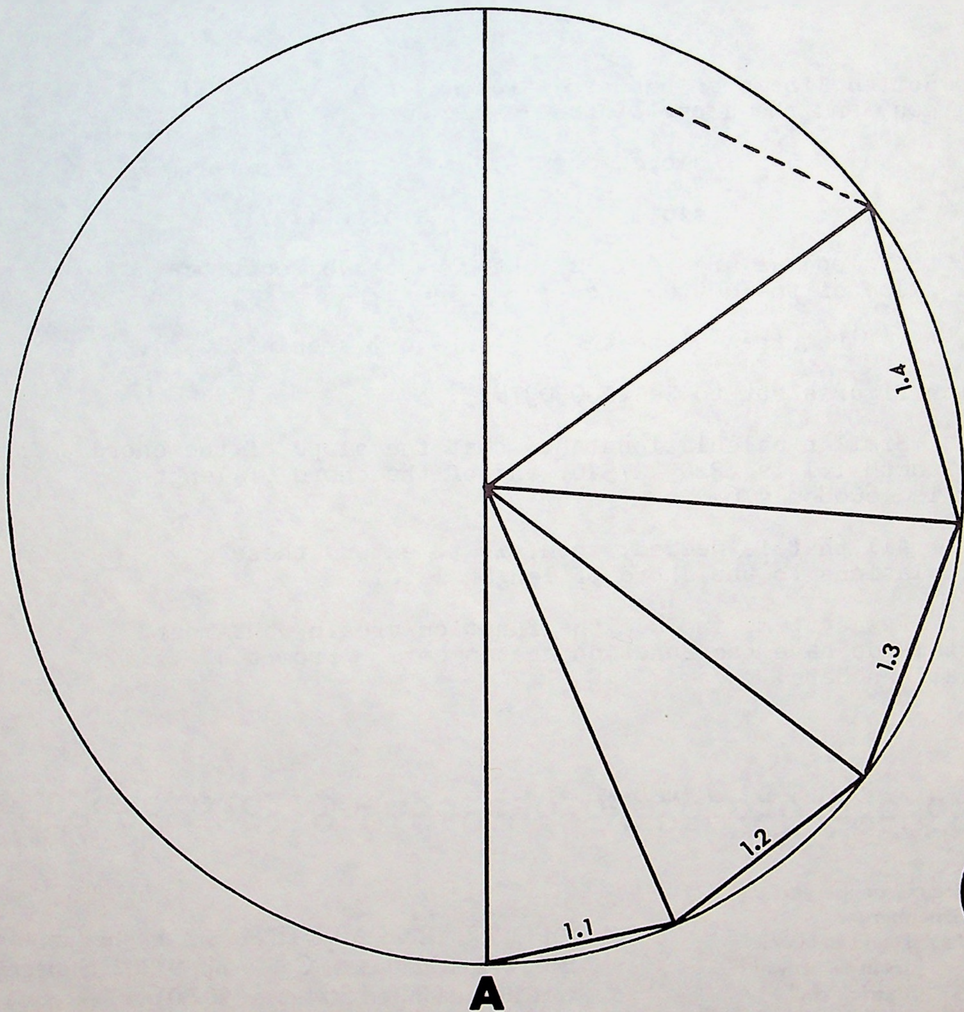
Popular Computing

81

The world's only magazine devoted to the art of computing.

December 1979

Volume 7 Number 12



F

The Last Chord

Here's an opportunity to demonstrate your ability to use BASIC (or whatever language you prefer) on a simple, straightforward scientific problem.

Given a circle 10 units in diameter, as in Figure F. Starting at the point marked A, successive chords are drawn, of lengths 1.0, 1.1, 1.2, 1.3,... units. There will be just 91 of them, the last one being a diameter of the circle. The Problem is: What is the <u>slope</u> of that last chord?

To find the slope of the first chord, see the exaggerated drawing of Figure G. The chord of length 1.0 subtends angle β. Angle θ is half of β. We have:

$$\text{sine } \theta = .5/5 = .10$$

$$\theta = \text{arcsine}(.10).$$

The dotted line then has direction given by $(-90 + \theta)$, in degrees, and the slope of the dotted line is then:

$$\text{slope} = \tan(-90 + \theta)$$

$$\text{slope} = \tan(-90 + \arcsin(.10))$$

and the slope we are seeking is the negative reciprocal of the slope of the dotted line.

$$\text{required slope} = -1/(\tan(-90 + \arcsin(.10))),$$

which figures out to be .1005037815.

Similar calculations show that the slope of the chord of length 1.1 is .3209427526, and of the chord of length 1.2 is .6008861476.

All that is needed, then, is to extend these calculations to the chord of length 10.0.

Few systems include the function arcsin, but most systems do have the function arctangent. From the relations here:

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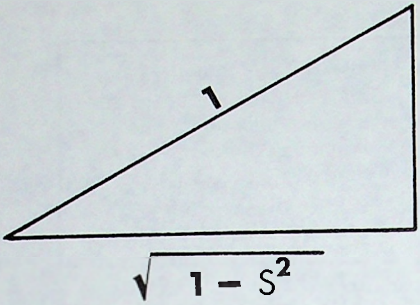
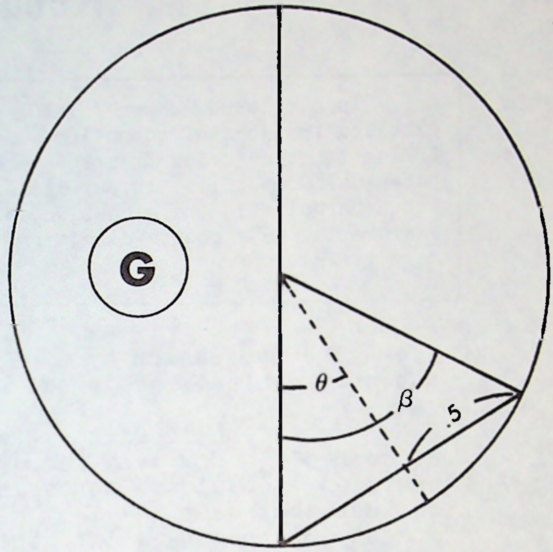
POPULAR COMPUTING is published monthly at Box 272, Calabasas, California 91302. Subscription rate in the United States is $20.50 per year, or $17.50 if remittance accompanies the order. For Canada and Mexico, add $1.50 per year. For all other countries, add $3.50 per year. Back issues $2.50 each.



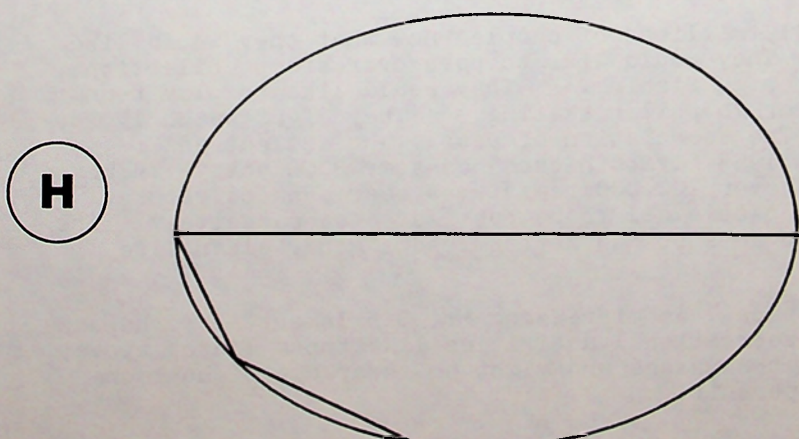


we can see that the angle whose sine is S is the same as the angle whose tangent is S divided by the square root of (1-S squared).

A casual calculation (made in an airport) with a pocket calculator gave these results: a total revolution of 6230.63 degrees for all 91 chords, with the last one then having a slop of +.376324. My confidence in these results is very low.

That was problem A. Now for problem B.

The circle is replaced with an ellipse (Figure H) whose major axis is 10 units long and whose minor axis is 6 units long. We start at one end of the major axis and draw chords of length 1.0, 1.1, 1.2, and so on, much as before. What is the length of the last chord? How many chords will there be? What is the slope of the last chord?



Rebuttal

In our issue number 78, the essay "Numbers" by Norman Sanders extolled the joys of computing. Apparently, Sanders' paean triggered the antithetical view from the anonymous "A. Cynic" who produced the harangue "Computniks" in our issue number 80. Perhaps "Numbers" was too hyperbolic in one direction, but "Computniks" surely goes to an extreme in the opposite direction. A rebuttal is called for, and here it is.

--fg

The profession by Cynic must be answered. He is not only cynical; he is extremely bitter--and he is wrong.

For if he is right, then nearly all of society stands condemned. How many people do you know who devote their energies and talents to paying their debt to society? Who defines that debt? They must never relax, even for a moment, or they fall into the parasitic class that Mr. C abhors. For if they relax, say to watch a game on TV, or to read a novel, or--horrors!--to do some problem solving with a computer, then they are draining society. Unless I have seriously misinterpreted the message, this is all errant, and arrogant, nonsense.

Usefulness lies in the mind of the practitioner. I personally can think of no useful output from all of astronomy, for example, which is not to condemn the pursuit of astronomy in any way. I find the recent information about the nature of Saturn and its moons to be of no conceivable use to me or, for that matter, to Mr. C. But that does not give me the right to recommend that the study of such things be outlawed. How in the world does one judge such things? And who appoints the judges?

Humans have dreamed for aeons of some day having machines to perform all the tedious work in the world, so that men could be made free--for what? Suppose that all food, clothing, and shelter could be supplied to everyone --then what would people do?

Well, millions of people know what they would like to do. They would like to pour over stamp collections, or bowl, or go fishing. They would like to play tennis, or golf, or go roller skating. They might watch TV, or read. The association of people who collect and refurbish 1953 Plymouth cars has some 3000 active members. There are over 100,000 registered beer can collectors. Are these people all to be castigated as parasites? The person who does so had better lead a good, clean life.

Perhaps I am misreading Mr. C's lament. Perhaps he condones recreational activities like those listed above, at least for the masses who might be incapable of any more "useful" pursuit.

What is a cynic? A man who knows the price of everything and the value of nothing.

--Oscar Wilde

Cynic, n, a blackguard whose faulty vision sees things as they are, not as they ought to be.

--Ambrose Bierce

It will generally be found that those who sneer habitually at human nature, and affect to despise it, are among its worst and least pleasant samples.

--Charles Dickens

The cynic is one who never sees a good quality in a man, and never fails to see a bad one.--He is the human owl, vigilant in darkness and blind to light, mousing for vermin, and never seeing noble game.

--Henry Ward Beecher

To admire nothing is the motto which men of the world always affect.--They think it vulgar to wonder or be enthusiastic.--They have so much corruption and charlatanism, that they think the credit of all high qualities must be delusive.

--Samuel Brydges

Mr. C's opinion of what intrigues people about computing doesn't agree with my experience. As I see computists (I see no point in his using denigrating terms like computniks) I see enthusiasts whose enjoyment comes from competent, well-planned, top-quality computing. Much of the fascination of getting the machine to perform correctly lies in the elaborate detective work of debugging and testing.

My experience also includes the feeling that almost any computing--even that done for sheer fun--tends to improve one's ability to do computing. Indeed, from the early issues of POPULAR COMPUTING we have promoted the maxim "The way to learn computing is to compute."

This matter of what is useful in the world--what is worth doing--what might constitute a drain on society's resources--what should be banned or forbidden--is not new, nor is it likely to be settled casually. In a free society, it is largely a matter of individual judgement, and Mr. C's is no better than yours, or mine.

There is a large element of Puritanism in Mr. C's attitude, to the effect that anything that anyone does that provides pleasure for that person must be immoral, or sinful, or fattening. Since time immemorial, whenever a pleasurable activity was devised, a group sprang up to declare it sinful. It would seem that the computer has now spawned a modern version of Anthony Comstock, who devoted himself (circa 1873) to suppressing what <u>he</u> considered immoral.

Somehow, I can occupy myself for hours on end with a computer and not feel one bit guilty, or parasitic. I can only look with great envy at the recent work of Harry Nelson and David Slowinski in establishing that

$$2^{44497} - 1$$

is prime, the largest now known--a record that will probably stand for several decades. My feeling about such "useless" work is a matter of taste; one should keep in mind the admonition de gustibus non est disputandum, else one may wind up with egg on one's face. As, for example, the professor (of Astronomy!) who wrote (in the Los Angeles Times, June 8, 1979):

> The largest prime number, which has 13,395 digits, was discovered by two computer experts at Livermore. This is the climax of three months of labor by two scientists using one of the most advanced computers available.
>
> Now that we have at our disposal a 13,395-digit prime number, what are we supposed to use it for? And what is the practical significance of this discovery?
>
> The answer to both questions is "nothing."
>
> Except for the pleasure of doing something that has not been done before, this is a mere waste of money and effort at a time when all levels of education in California are suffering from severe budget cuts.
>
> PROF. S. I. SALEM
> Chairman-Elect
> Department of Physics-Astronomy
> California State University
> Long Beach

Mr. C must be interested in something besides his day-to-day work, or else he would be labelled a very dull boy indeed. It is interesting to speculate on just what he can find to do with his time that would satisfy his own standards of usefulness and worthwhileness. Let's see: we can rule out art and music (what could be more useless?). Besides, enjoying music without producing some of it would be parasitic. He must not collect anything--collectors are the apotheosis of parasitism. He must not engage in any activity that amuses him or does no harm, lest he inadvertently incur someone else's scorn. He must certainly not do anything that might be noteworthy, since it might then come to the attention of the Guiness people. Good grief!--what does he do with his spare time?

Book Review...

Computer Games (for Businesses, Schools, and Homes)

by J. Victor Nahigian and William S. Hodges

Winthrop Publishers, 1979, 8 1/2 x 11 paper cover, 157 pages, $10.95.

Here is another book full of canned BASIC programs, 27 of them this time, and labelled "Games." That term is used loosely, since a program to calculate the dates of Easter is not a game in the usual sense.

The problem of portability of the programs was handled here again by attempting to use only the most elementary of BASIC constructs. However, the authors were using a time-shared version of BASIC (DEC PDP8/I), with output to a Teletype. And right away there is trouble. For one thing, their Teletype didn't slash either the oh or the zero. Their system used a back-slash (\) to separate multiple statements on the same line; my BASIC doesn't even have a back-slash. Their BASIC and mine differ in the use of the RND function. Some of the programs use double subscripts, which many current BASICs do not permit. They use variable names of the form LD (L for a letter; D for a digit), whereas that constraint was relaxed for most BASICs long ago.

And so it goes--until there is some effort toward standardizing BASIC, any author is in a no-win situation.

If the purchaser of this book wants only to key the programs into his machine (with all the necessary changes to get it to run in his system) in order simply to play the games--that is, if he has no interest in doing any computing or programming, then there will probably be little harm done. But wait!--the Foreword, by Gerald Weinberg, says:

> Readers who work with the game programs will eventually become impatient with the restrictions of BASIC and the narrowness of the world of games. When that happens, Computer Games...will have achieved its greatest triumph--starting people soundly on the road to creative, compassionate programming.

...so there is some intention to foster good computing. Try this as an example of how to foster bad computing:

```
400 GO TO 410
410 PRINT
```

(this on page 45 of the book). The programs in the book are loaded with GOTOs; they are fine examples of what Weinberg himself calls spaghetti programs. (In all fairness, the program on page 45, to calculate the dates of Easter, calculates correctly. But there is not the faintest clue as to what is being done, or how, or why; that is, no suggestion as to how the reader might improve the program, or alter it to fit other purposes.)

It would appear, then, that the authors accumulated some BASIC programs, and Winthrop packaged them for sale. It's no big deal, but it's sad to see. So many opportunities were passed up to foster some decent computing. For example, their game of ROLL ON is a dice game (described in our issue Number 7) and, judging by their sample run, it plays properly. But (1) the code cries out to be cleaned up, and (2) it would be so nice to suggest how to use the program to develop some strategy of winning. Oh well, why do I bother?

There is, of course, a version of Star Trek included.

The center two pages contain a flowchart involving three switches.

A switch is a programming device for storing a decision. For example, switch A has three possible exits, labelled A1, A2, and A3, but the choice of which exit is operational is made elsewhere in the logic.

All that we are doing is manipulating the three switches, and counting the number of times that each of the 12 paths is traversed.

A computer program to perform these manipulations can be written for any computer ever built, in any known language. The logic could be implemented on a programmed calculator.

Any beginning student ought to be able to write a program to follow the logic of the flowchart.

The whole thing is absurdly simple. The program is to be written to halt when counter 12 reaches some limit, say 50. At that point, the other eleven counters can be examined.

And now for the Problem:

Will counter 12 ever reach 50?

Or reach one, for that matter?

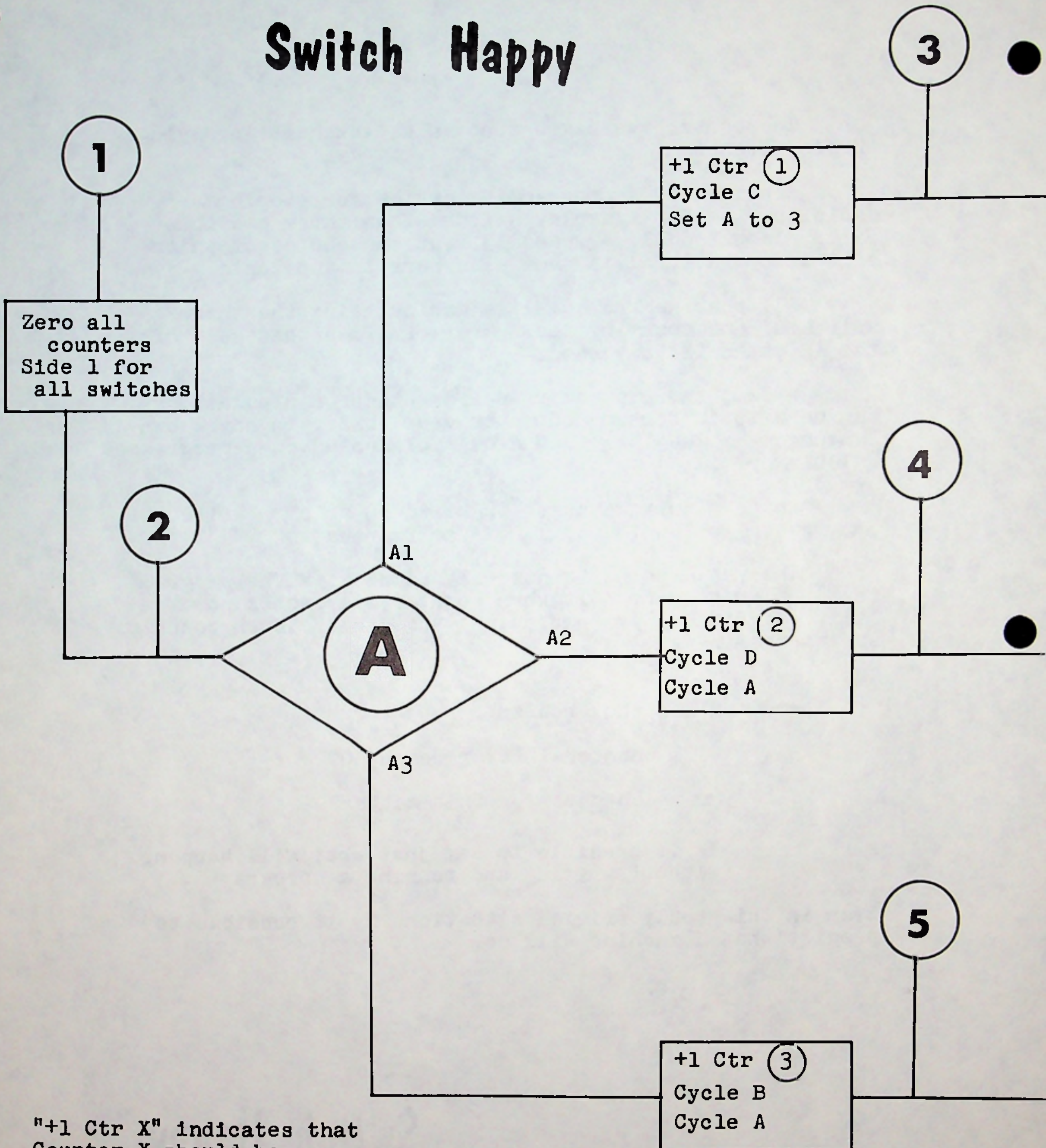
Is it possible to say just what will happen, without writing and running a program?

Even in this truly trivial situation, is it possible to predict what a machine will do?

PROBLEM **265**

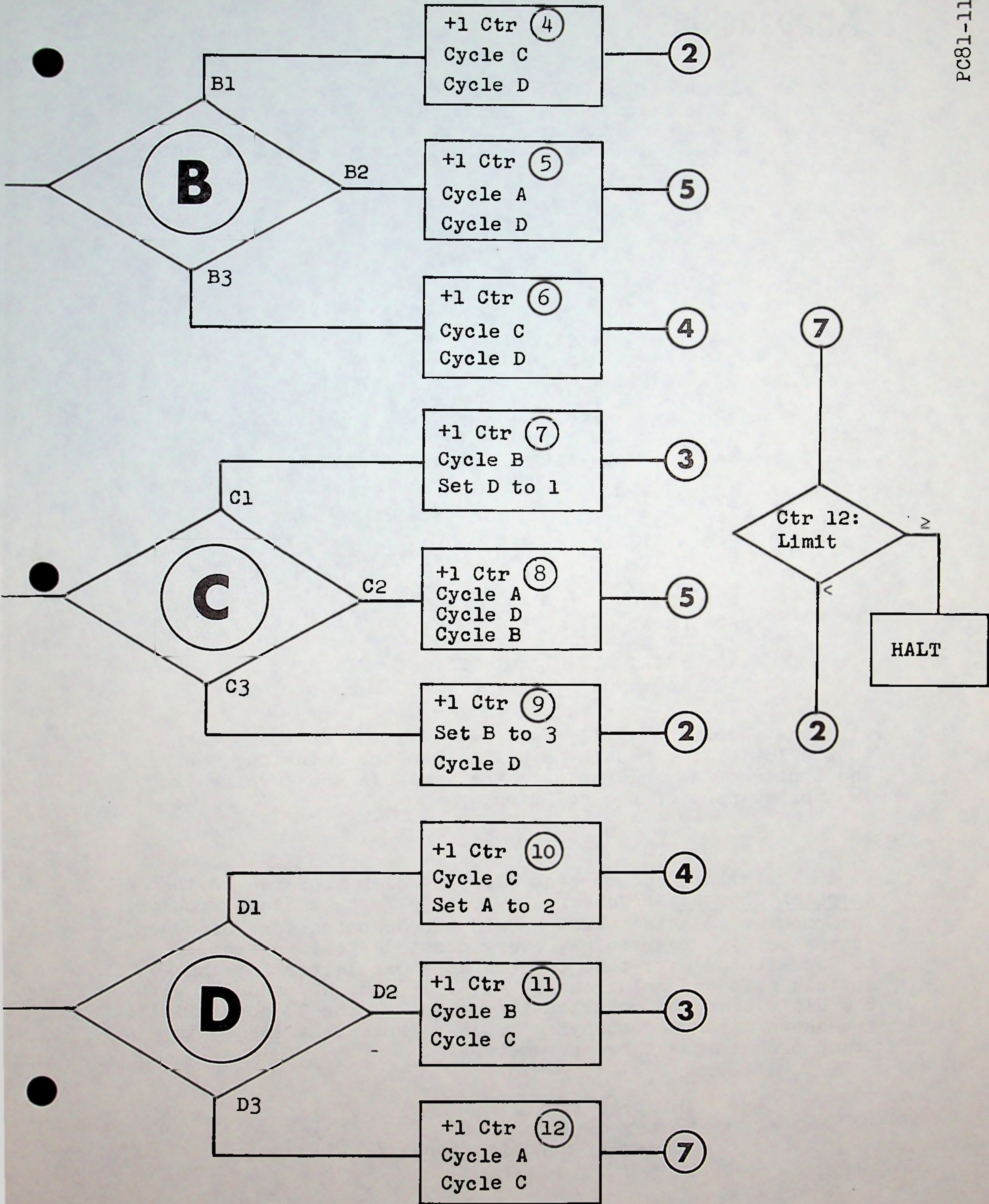
Switch Happy

Switch Happy



"+1 Ctr X" indicates that Counter X should be incremented by one.

"Cycle" means to advance the switch in cyclic order; that is, from position 1 to position 2, or 2 to 3, or 3 to 1.


B
B1
+1 Ctr 4
Cycle C
Cycle D
2
B2
+1 Ctr 5
Cycle A
Cycle D
5
B3
+1 Ctr 6
Cycle C
Cycle D
4
C
C1
+1 Ctr 7
Cycle B
Set D to 1
3
C2
+1 Ctr 8
Cycle A
Cycle D
Cycle B
5
C3
+1 Ctr 9
Set B to 3
Cycle D
2
D
D1
+1 Ctr 10
Cycle C
Set A to 2
4
D2
+1 Ctr 11
Cycle B
Cycle C
3
D3
+1 Ctr 12
Cycle A
Cycle C
7
7
Ctr 12:
Limit
≥
HALT
<
2

Knapsacking

Problem 48 (BULLSEYE) in the book by Spencer (see the review elsewhere in this issue) asks how to score exactly 100 with 6 shots at a target whose circles count 16, 17, 23, 24, 39, and 40. The solution given (slightly modified) is:

```
10  A = 40
20  B = 39
30  C = 24
40  D = 23
50  E = 17
60  F = 16
100 FØR A1 = 0 TØ 2
110 FØR B1 = 0 TØ 2
120 FØR C1 = 0 TØ 3
130 FØR D1 = 0 TØ 4
140 FØR E1 = 0 TØ 5
150 FØR F1 = 0 TØ 6
200 G = A1*A+B1*B+C1*C+D1*D+E1*E+F1*F
210 IF G = 100 THEN 400
220 NEXT F1
230 NEXT E1
240 NEXT D1
250 NEXT C1
260 NEXT B1
270 NEXT A1
300 PRINT "FAILURE"
310 GØTØ 600
400 PRINT A1, B1, C1, D1, E1, F1
600 END
```

with the result: four 17's and two 16's. It would seem that a lot of the result is built into the solution; that is, the choice of a range of 6 on the variable whose value is 16 implies part of the solution.

A similar, but more complex, problem appeared in the Computer Journal, November 1969, as "A Postage Stamp Problem." The problem is this: What should the denominations of seven stamps be, in order to pay every possible postage from 1¢ to 70¢ with no more than three stamps per letter? The article gave the solution: 1, 4, 5, 15, 18, 27, and 34, with the claim that the solution is unique. The 70 possibilities are shown in an accompanying table. Note that the Postage Stamp problem has three parameters:

An upper limit, L	(70)
Number of stamps, S	(3)
Number of denominations, K	(7)

1	1		
2	1	1	
3	1	1	1
4	4		
5	4	1	
6	5	1	
7	5	1	1
8	4	4	
9	5	4	
10	5	5	
11	5	5	1
12	4	4	4
13	4	4	5
14	5	5	4
15	15		
16	15	1	
17	15	1	1
18	18		
19	18	1	
20	18	1	1
21	15	5	1
22	18	4	
23	18	4	1

24	15	5	4
25	15	5	5
26	18	4	4
27	27		
28	27	1	
29	27	1	1
30	15	15	
31	15	15	1
32	27	5	
33	18	15	
34	34		
35	34	1	
36	34	1	1
37	27	5	5
38	34	4	
39	34	4	1
40	18	18	4
41	18	18	5
42	34	4	4
43	34	4	5
44	34	5	5
45	27	18	
46	27	18	1

47	27	15	5
48	18	15	15
49	34	15	
50	34	15	1
51	18	18	15
52	34	18	
53	34	18	1
54	27	27	
55	27	27	1
56	34	18	4
57	34	18	5
58	27	27	4
59	27	27	5
60	27	18	15
61	34	27	
62	34	27	1
63	27	18	18
64	34	15	15
65	34	27	4
66	34	27	5
67	34	18	15
68	34	34	
69	34	34	1
70	34	18	18

The original Postage Stamp problem. The seven denominations (1, 4, 5, 15, 18, 27, 34) are uniquely determined, but the solution above is just one possible arrangement. 37¢ postage could be made with 18, 15, and 4, for example.

Currently, another version of the same problem is getting attention; namely, the knapsack problem (see "The Mathematics of Public-Key Cryptography" by Martin Hellman, Scientific American, August 1979). As Hellman puts it:

> Given a set of numbers $a_1, a_2, \ldots, a_n$ and a sum C, determine which of the numbers add up to C.

In the BULLSEYE problem and the Postage Stamp problem, repetitions of the numbers a_k are permitted; it is not clear from the above statement whether or not repetitions are permitted in the ordinary knapsack problem.

All of these problems are in the class called NP (for nondeterministic, polynomial time), and the Postage Stamp problem is NP-hard.

The NP problems have this common characteristic: they are all computationally fierce (Professor Don Knuth applied the terms "Herculean," "Formidable," and "arduous" to them) but any result can be quickly verified. For example, we can easily postulate a more complex version of the Postage Stamp problem: we can select any two of the three parameters arbitrarily, say,

$$L = 123$$
$$S = 5$$

and make a stab at

$$K = 9$$

(there is no way to tell, a priori, whether 9 denominations are sufficient, or overly-abundant). But now if someone produces a solution for this case, we can readily verify that it is correct.

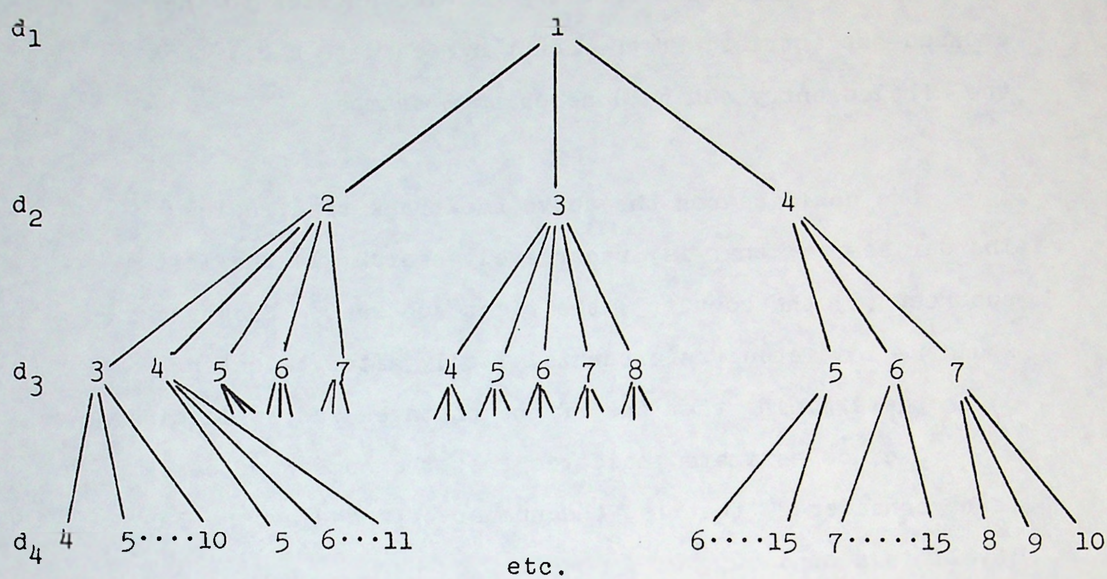
Some preliminary considerations come from Prof. Robert Henderson of the Mathematics Department of California State University, Northridge:

As a programming problem, the Postage Stamp problem would seem amenable to backtracking. Suppose we consider generating the denominations in increasing order:

$$d_1 < d_2 < d_3 < d_4 < d_5 < d_6 < d_7$$

Clearly, $d_1 = 1$ and d_2 must be one of 2, 3, or 4.

Continuing in this fashion we get the following tree. Each level corresponds to a subscript of d (7 levels):



Having chosen $d_1, d_2, \ldots, d_k$, then d_{k+1} has a possible range of values from d_k+1 up to the first postage value which can't be produced from 3 stamps taken from $\{d_1, d_2, \ldots, d_k\}$. This would seem to be the most annoying part of the programming. It might be improved a bit (speedwise) by forming an intermediate list of the values that can be obtained using 2 stamps. That is, with

$d_1 = 1,\ d_2 = 4,\ d_3 = 5,\ d_4 = 15,\ d_5 = 18,\ d_6 = 27,\ d_7 = 34$

we get this pattern:

~~1~~	~~2~~	3	~~4~~	~~5~~	~~6~~	7	~~8~~	~~9~~	~~10~~
11	12	13	14	~~15~~	~~16~~	17	~~18~~	~~19~~	~~20~~
21	~~22~~	~~23~~	24	25	26	~~27~~	~~28~~	29	~~30~~
~~31~~	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	40
41	~~42~~	43	44	~~45~~	46	47	48	~~49~~	50
51	~~52~~	53	~~54~~	55	56	57	58	59	60
~~61~~	62	63	64	65	66	67	~~68~~	69	70

/ denotes a value that can be obtained using 2 stamps.

◯ denotes a value that can't be so obtained.

then one can count back from the circled entries to the crossed-out entries; if the count agrees with a d_i, then the circled entry can be done using 3 stamps.

One could search the above backtrack tree, using inorder search; that is, recursively search (1) the left subtree, (2) the root, (3) the right subtree. Backtracking would be initiated (unfortunately) only after selecting d_7 and determining that one of the postages was still missing.

A crude estimate indicates that the number of cases to be considered (that is, the number of nodes at the 7th level) is around 80,000.

Herman P. Robinson, Lafayette, California, attacked the Postage Stamp problem, and reports as follows:

First, it is necessary to start with a set of stamps $s_1, s_2, \ldots, s_n$ and calculate all possible sums using 1, 2, or 3 stamps. Here, $s_1 = 1$. These sums are arranged in order and the largest one before a gap in the list is considered the maximum P for that set. A new set is chosen and a new P obtained. Whenever a P is obtained that is equal to or larger than the previous one, it is stored with the particular set that produced it. The previous P is also stored, so that at the end of all calculations, it can be noted whether the final set is unique or not. In general, calculations should start with

1, 2, 3,...,n.

There are some conditions that can speed up the calculation. When the term s_i is larger than $3s_{i-1} + 1$, there is no way to produce the sum $3s_{i-1} + 1$, so the need to increase the last term is terminated. The previous term is then increased, and it is stopped when $3s_{i-2} + 1$ is greater than s_{i-1}, etc. We shall call a condition that stops a set from giving new P's a locked condition. When s_2 reaches a value of 5, all possible combinations of n stamps have been tested and their P's calculated.

There may be other conditions causing a lock. For example, the set 1, 4, 7 is the last point before being locked because when the last stamp is 8 or greater, there is no way to produce a 7. In this case, the 4 would be raised to 5, but that ends all the calculations because there is no way to produce a sum of 4. It is desirable to have a method of finding a locked condition, but I don't know of one. It is true that in a locked condition, the P's will all be the same. Unfortunately, this can happen for a limited number of P's when there is no locked condition. However, when the set is not too large (n is small), then a few P's the same will indicate a lock. If two equal P's are tested for, we shall call it 2-lock; four P's, 4-lock, and no test for same P's, no-lock. This last assures complete accuracy. A list of some of the lock positions would be useful for the computer.

We can start with n = 1:

s_1	possible postages	P
1	1,2,3	3

$s_1 s_2$	possible postages	P
1,2	1,2,3,4,5,6	6
1,3	1,2,3,4,5,6,7	7
1,4	1,2,3,4,5,6	6
1,5	1,2,3	3

P	sequence							
9	1	2	3					
10	1	2	4					
12	1	2	5					
10	1	2	6					
11	1	2	7					
6	1	2	8					
12	1	3	4					
11	1	3	5					
10	1	3	6					
11	1	3	7					
12	1	3	8					
7	1	3	9					
7	1	3	10					
15	1	4	5	*				
14	1	4	6					
99	1	4	7					
6	1	4	8					
6	1	4	9					
6	1	4	10					
12	1	2	3	4				
13	1	2	3	5				
15	1	2	3	6				
17	1	2	3	7				
14	1	2	3	8				
15	1	2	3	9				
16	1	2	3	10				
9	1	2	3	11				
9	1	2	3	12				
24	1	4	7	8	*			
36	1	4	6	14	15	*		
52	1	4	6	14	17	29	*	
38	1	4	5	15	18	27	28	
52	1	4	5	15	18	27	29	
43	1	4	5	15	18	27	30	
43	1	4	5	15	18	27	31	
38	1	4	5	15	18	27	32	
43	1	4	5	15	18	27	33	
70	1	4	5	15	18	27	34	*
51	1	4	5	15	18	27	35	
38	1	4	5	15	18	27	36	
43	1	4	5	15	18	27	37	
51	1	4	5	15	18	27	38	
51	1	4	5	15	18	27	39	
79	1	4	5	15	16	18	24	37

An asterisk indicates a unique solution, with the possible exception of $n = 6$.

H. P. Robinson
23 July 1979

Additional results are shown in the accompanying table. Sets for n = 3, 4, and 5 were calculated for 2-lock, 4-lock, and no-lock, with all results the same. For n = 5, 4-lock took 40 minutes and no-lock took over 4 hours, showing the value of some kind of lock test. The n = 7 set was not calculated except for a few cases around the maximum given in the Computer article. These show that a 2-lock would have failed to find the right answer because of the two P = 43's just before the answer. A partial calculation was made for n = 8. It was started at 1, 4, 5, 6, 7, 8, 9, 10 because all the other optimum sets occur at 1, 2,... or 1, 3,... An expert in number theory might help here. The Wang ran for 24 hours using 4-lock, and showed that with 8 different stamps one has a P of at least 77. One set for this would be 1,4,5,6,9, 21, 32, 34 and another 1,4,5,6,10,21,32,35, and I think there is at least one more that gives 77. My guess is that the maximum P occurs for 1,4,5,15,... I jumped the calculation to start at 1,4,5,15,16,17,18,19 and soon found a P of 79 at 1,4,5,15,16,18,24,37. P max is probably much greater than 79. I hope you can find it.

At the present time, then, it looks as if the optimum stamp arrangements give the following results:

n	P
1	3
2	7
3	15
4	24
5	36
6	52
7	70
8	>79

From time to time we neglect to label our problems with the consecutive numbering scheme that we have used over the years:

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Book Review...

<u>Sixty Challenging Problems with BASIC Solutions</u>

by Donald D. Spencer

Hayden Books, 1979, 6 x 9 paper cover;
128 pages, $6.95.

Mr. Spencer is probably competent to write an intelligent computer program, even in BASIC. What he actually does is whip off the type of program that undergraduates (or high school students) delight in, categorized by the word "slapdash." He has a great interest in old puzzle problems, and he finds publishers who have a great interest in publishing slapdash books.

Back in the olden days, the appearance of a "second edition" meant (besides brisk sales) that the minor troubles of the first edition had been uncovered and cleared up. Today, apparently, it means the same as "second printing" only with a different color cover.

Consider, for example, problem number 4, "Factoring Numbers." The statement of the problem contains a serious error, and the solution given contains a few more. The method (attributed to Fermat) assumes that the number being factored is odd (a reasonable assumption; who tries to factor even numbers?). The second example given is N = 368, for which the book's program prints

THE LARGEST FACTOR OF 368 IS 8.

But even if the number to be factored <u>is</u> odd, the program works on some numbers but goes into an endless loop on others.

Each of the 60 problems has a solution in the form of a BASIC program. But <u>which</u> BASIC? To be sure, the programs use only the most elementary BASIC statements, but nevertheless the reader should be warned that the programs will probably not execute properly in the BASIC they have without some tinkering.

The name of Eratosthenes is misspelled six times. This is a second edition?

The BASIC programs evidently accumulated over some period of time. A number is squared sometimes with W*W, sometimes with W^2. The oh and the zero are crossed or not crossed somewhat at random. Numbers to be supplied to a program are sometimes INPUT, sometimes LET, sometimes READ, with no common scheme.

Pages 10 through 84, which contain the 60 problem statements, also contain cartoons, to the extent of 50% of the space. Thus, the book has only 81 pages of useful material in it, which makes it pretty darn expensive.